Activity description

In this activity students create spreadsheets to model what would happen to the temperature of the Earth if there were to be a sudden change in the amount of radiation entering or leaving the planet.

Students then investigate polynomial and exponential functions to find the best model.

Suitability

Level 3 (Advanced)

Time

1–2 hours

Resources

Student information sheet
Excel spreadsheet
Optional: slideshow

Key mathematical language

Model, constants, initial conditions, function, quadratic, cubic, quartic, exponential.

Notes on the activity

This version of the activity, Climate A, assumes students have access to Excel spreadsheets. Climate B is an alternative paper-based activity requiring use of a graphic calculator.

Either version can be used independently, or students can use both – this would allow them to compare the methods used.

Before doing this activity students will need to have learnt how to use a spreadsheet to:

- write spreadsheet formulae involving both relative and absolute cell references
- draw graphs
- use trendlines to find polynomial regression lines.

Students will also need to know how to enter numbers in standard form into a spreadsheet.

If students already know how to substitute data values into exponential functions and solve the resulting equations, they could be asked to find their own exponential functions to model the temperature data.

During the activity

The work may be shared between students if you wish.

Points for discussion

Check that students can calculate the outgoing radiation = σT^4 = 364 J $s^{-1}m^{-2}$ (to 3sf), and that they know the connection between Kelvin and Celsius temperatures.

Discuss, in general terms, how students expect the Earth's temperature to change after a sudden increase in radiation input.

At the end of the activity, discuss why an exponential function gives a better long-term prediction than any of the polynomials.

Extensions

More able students could devise their own spreadsheets.

If students have time, they could find models for other percentage increases and decreases.

Students could try using different time increments.

They could investigate an ongoing small percentage increase in radiation input – or decrease in radiation loss.

Climateprediction.net: information and acknowledgement

This is an adaptation of an activity written by Sylvia Knight (University of Oxford Atmospheric, Oceanic and Planetary Physics) and Jon Gray (Banbury School) for a Nuffield project linked to the climateprediction.net project.

Although this activity does not require that students have any detailed knowledge of climate or of the climateprediction.net project, they could be encouraged to find out more by visiting www.climateprediction.net.

Further activities linked to the project and information for teachers are also available from www.climateprediction.net/schools.

Climateprediction.net is a joint research project funded by the Natural Environment Research Council (NERC) and the Department of Trade & Industry. Its aim is to use the large number of idle computers worldwide and the power of the internet to predict and understand the climate. Your students can find out more and take part by visiting www.climateprediction.net and downloading their own unique simulation model of the Earth's climate. The downloaded program runs as a background process (it does not affect normal computing) to generate data for a climate model. The graphics packages supplied with the model show how weather patterns develop. Results from these experiments were contributed to the 4th Assessment Report of the Intergovernmental Panel on Climate Change (IPCC) and will help policy makers plan for the effects of climate change.

Answers

5% increase

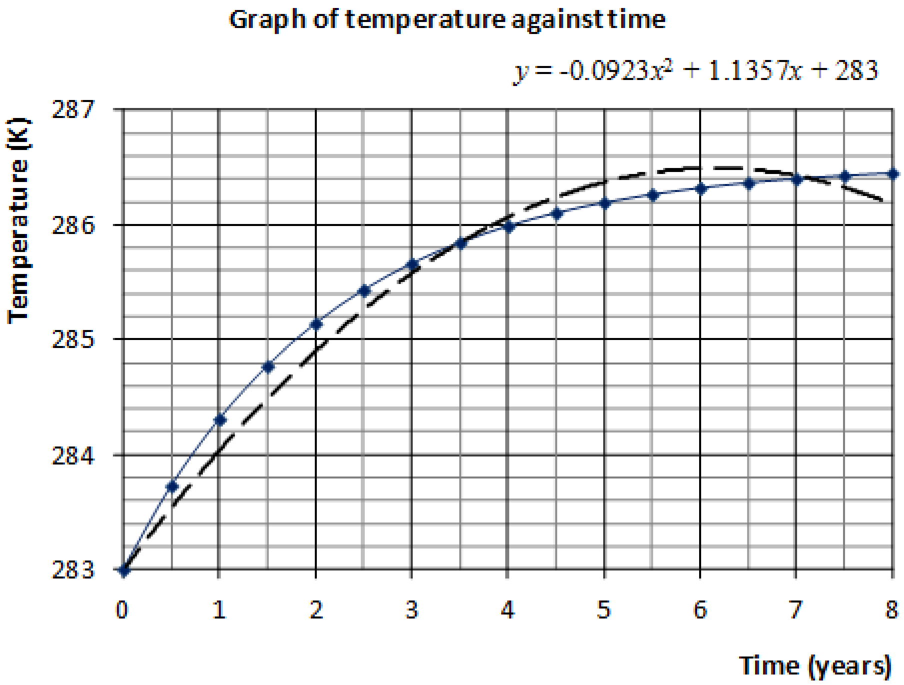
The way in which the temperature and outgoing radiation change with time is shown below.

	A	B	C	D	E
1	5.67E-08	400000000	15768000		
2					
3	**Time**	**Incoming**	**Outgoing**	**Temperature change**	**Temperature**
4	0.0	364	364		283
5	0.5	382.2	363.6878571	0.729748672	283.7297487
6	1.0	382.2	367.4536387	0.581301562	284.3110502
7	1.5	382.2	370.4742413	0.462229406	284.7732796
8	2.0	382.2	372.889373	0.367024918	285.1403046
9	2.5	382.2	374.8154596	0.291098583	285.4314031
10	3.0	382.2	376.3483954	0.230670254	285.6620734
11	3.5	382.2	377.5664488	0.182654589	285.844728
12	4.0	382.2	378.5330513	0.144551119	285.9892791
13	4.5	382.2	379.299326	0.11434457	286.1036237
14	5.0	382.2	379.9062975	0.090417753	286.1940414
15	5.5	382.2	380.3867746	0.071477345	286.2655188
16	6.0	382.2	380.7669254	0.0564918	286.3220106
17	6.5	382.2	381.0675774	0.044640097	286.3666507
18	7.0	382.2	381.30528	0.035269861	286.4019205
19	7.5	382.2	381.493166	0.027863396	286.4297839
20	8.0	382.2	381.6416461	0.02201031	286.4517942

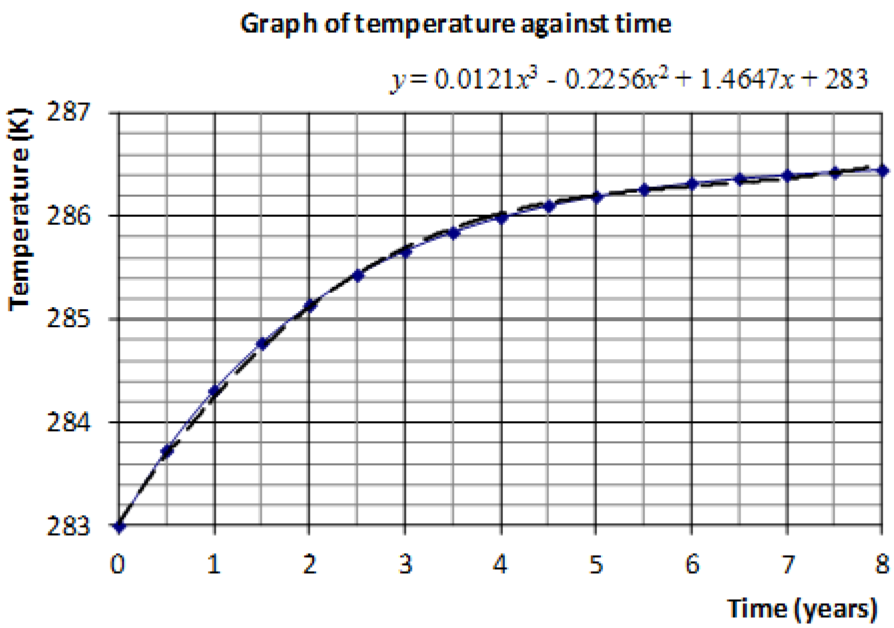
Formulae that can be used to generate these values are:

	A	B	C	D	E
1	=5.67*10^(-8)	=4*10^8	=182.5*24*60*60		
2					
3	**Time**	**Incoming**	**Outgoing**	**Temperature change**	**Temperature**
4	0	364	364		283
5	=A4+$C$1/(365*24*60*60)	=1.05*B4	=$A$1*E4^4	=(B5-C5)*$C$1/$B$1	=E4+D5
6	=A5+$C$1/(365*24*60*60)	=B5	=$A$1*E5^4	=(B6-C6)*$C$1/$B$1	=E5+D6
7	=A6+$C$1/(365*24*60*60)	=B6	=$A$1*E6^4	=(B7-C7)*$C$1/$B$1	=E6+D7
8	=A7+$C$1/(365*24*60*60)	=B7	=$A$1*E7^4	=(B8-C8)*$C$1/$B$1	=E7+D8
9	=A8+$C$1/(365*24*60*60)	=B8	=$A$1*E8^4	=(B9-C9)*$C$1/$B$1	=E8+D9
10	=A9+$C$1/(365*24*60*60)	=B9	=$A$1*E9^4	=(B10-C10)*$C$1/$B$1	=E9+D10
11	=A10+$C$1/(365*24*60*60)	=B10	=$A$1*E10^4	=(B11-C11)*$C$1/$B$1	=E10+D11
12	=A11+$C$1/(365*24*60*60)	=B11	=$A$1*E11^4	=(B12-C12)*$C$1/$B$1	=E11+D12
13	=A12+$C$1/(365*24*60*60)	=B12	=$A$1*E12^4	=(B13-C13)*$C$1/$B$1	=E12+D13
14	=A13+$C$1/(365*24*60*60)	=B13	=$A$1*E13^4	=(B14-C14)*$C$1/$B$1	=E13+D14
15	=A14+$C$1/(365*24*60*60)	=B14	=$A$1*E14^4	=(B15-C15)*$C$1/$B$1	=E14+D15
16	=A15+$C$1/(365*24*60*60)	=B15	=$A$1*E15^4	=(B16-C16)*$C$1/$B$1	=E15+D16
17	=A16+$C$1/(365*24*60*60)	=B16	=$A$1*E16^4	=(B17-C17)*$C$1/$B$1	=E16+D17
18	=A17+$C$1/(365*24*60*60)	=B17	=$A$1*E17^4	=(B18-C18)*$C$1/$B$1	=E17+D18
19	=A18+$C$1/(365*24*60*60)	=B18	=$A$1*E18^4	=(B19-C19)*$C$1/$B$1	=E18+D19
20	=A19+$C$1/(365*24*60*60)	=B19	=$A$1*E19^4	=(B20-C20)*$C$1/$B$1	=E19+D20

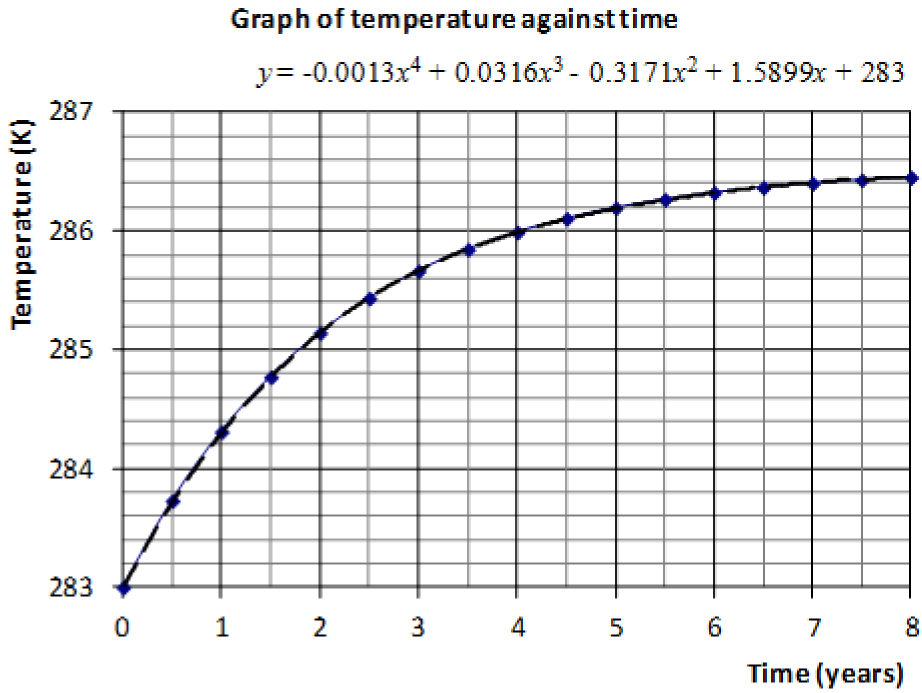
The following graph shows the way in which temperature changes with time as well as the **quadratic** function (dotted line) given by Excel to model the data:.



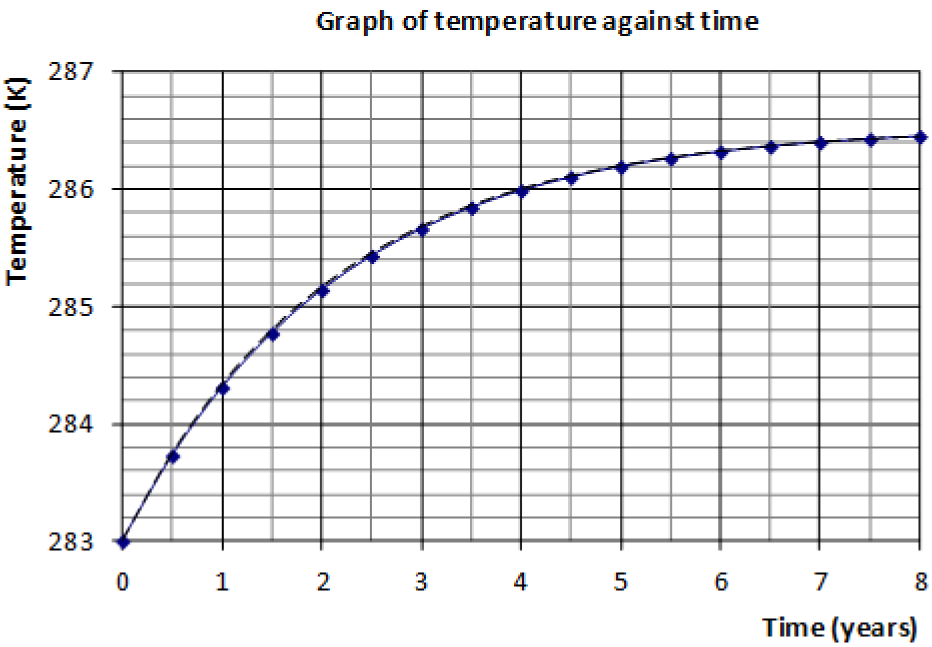
The next graph shows the **cubic** function (dotted line) given by Excel to model the data.



The next graph shows the **quartic** function (dotted line) given by Excel to model the data:



The **exponential** function $y = 283 + 3.53(1 - e^{-0.475t})$ is shown with the data below.



The graphs show that the functions
$y = 0.0121x^3 - 0.2256x^2 + 1.4647x + 283$,
$y = -0.0013x^4 + 0.0316x^3 - 0.3171x^2 + 1.5899x + 283$ and
$y = 283 + 3.53(1 - e^{-0.475t})$ all give values close to the temperature values for the first 8 years, but the values given by the quadratic function
$y = -0.0923x^2 + 1.1357x + 283$ are not so good.

If the graphs are extended to show later years, it can be seen that only the exponential function gives values that are likely to model what happens in practice. That is, the temperature approaches a new equilibrium value higher than the original 283 K.

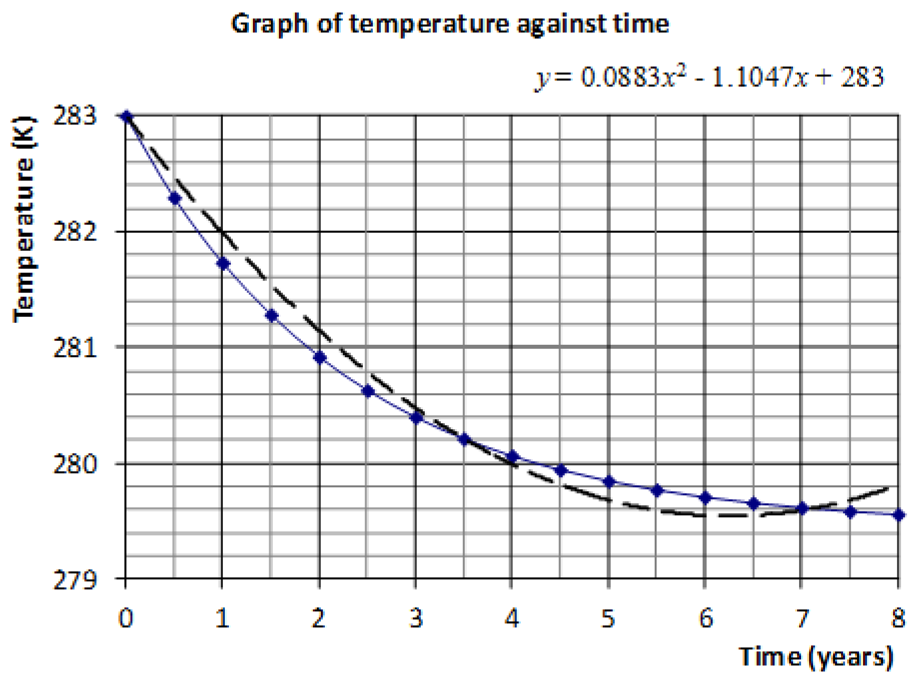
The quadratic model predicts a temperature fall after 6 years.
The quartic model predicts a temperature fall after 7 years.
The cubic model predicts a steep rise in temperature after 10 years.

5% decrease

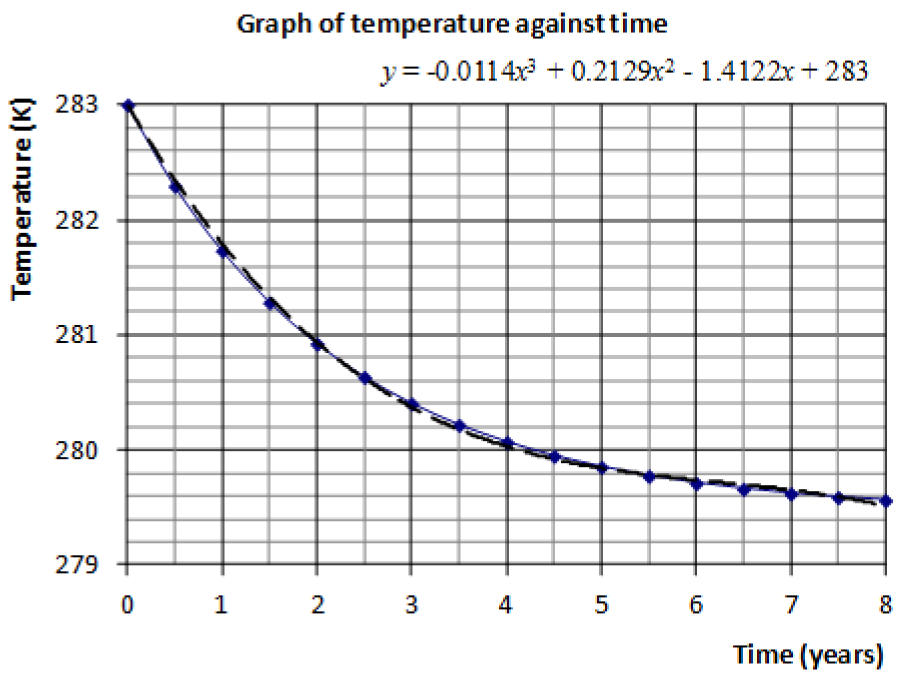
The way in which the temperature and outgoing radiation change with time after a 5% decrease in the incoming radiation is shown below. The values were obtained by replacing 1.05 by 0.95 in the formula in cell B5.

	A	B	C	D	E
1	5.67E-08	400000000	15768000		
2					
3	**Time**	**Incoming**	**Outgoing**	**Temperature change**	**Temperature**
4	0.0	364	364		283
5	0.5	345.8	363.6878571	-0.705139328	282.2948607
6	1.0	345.8	360.0766384	-0.562785088	281.7320756
7	1.5	345.8	357.2138081	-0.449932315	281.2821433
8	2.0	345.8	354.9373555	-0.360194553	280.9219487
9	2.5	345.8	353.1227911	-0.288664424	280.6332843
10	3.0	345.8	351.6736059	-0.231537545	280.4017467
11	3.5	345.8	350.5144429	-0.185843341	280.2159034
12	4.0	345.8	349.5861172	-0.149248738	280.0666547
13	4.5	345.8	348.841925	-0.119912684	279.946742
14	5.0	345.8	348.2448712	-0.096376824	279.8503652
15	5.5	345.8	347.76556	-0.077482374	279.7728828
16	6.0	345.8	347.3805755	-0.062306285	279.7105765
17	6.5	345.8	347.0712279	-0.050111805	279.6604647
18	7.0	345.8	346.8225753	-0.040309918	279.6201548
19	7.5	345.8	346.6226562	-0.032429107	279.5877257
20	8.0	345.8	346.4618851	-0.026091512	279.5616342

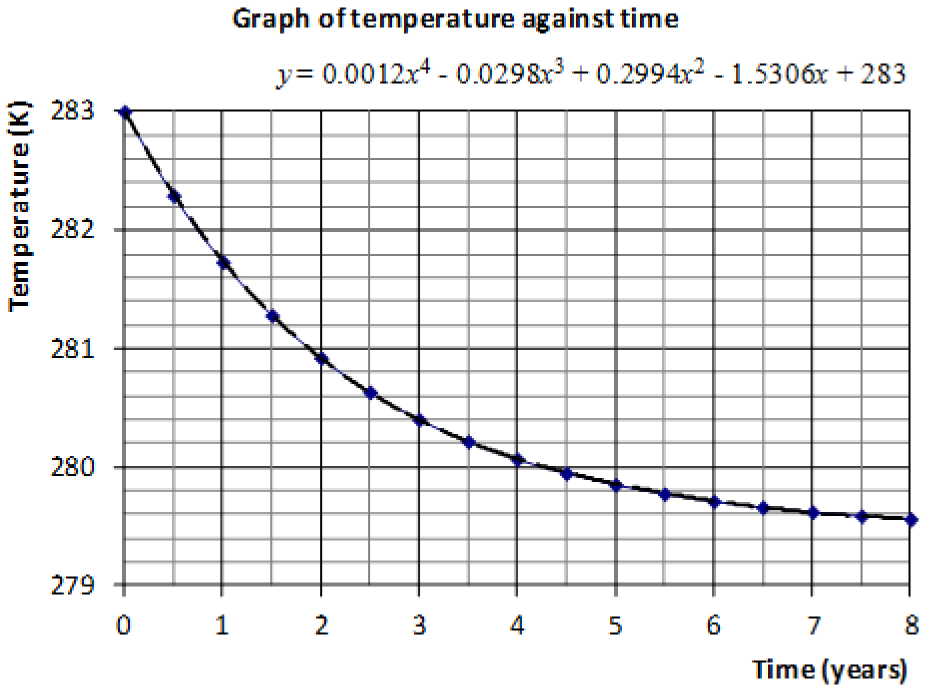
The following graph shows the way in which temperature changes with time, as well as the **quadratic** function (dotted line) given by Excel to model the data.



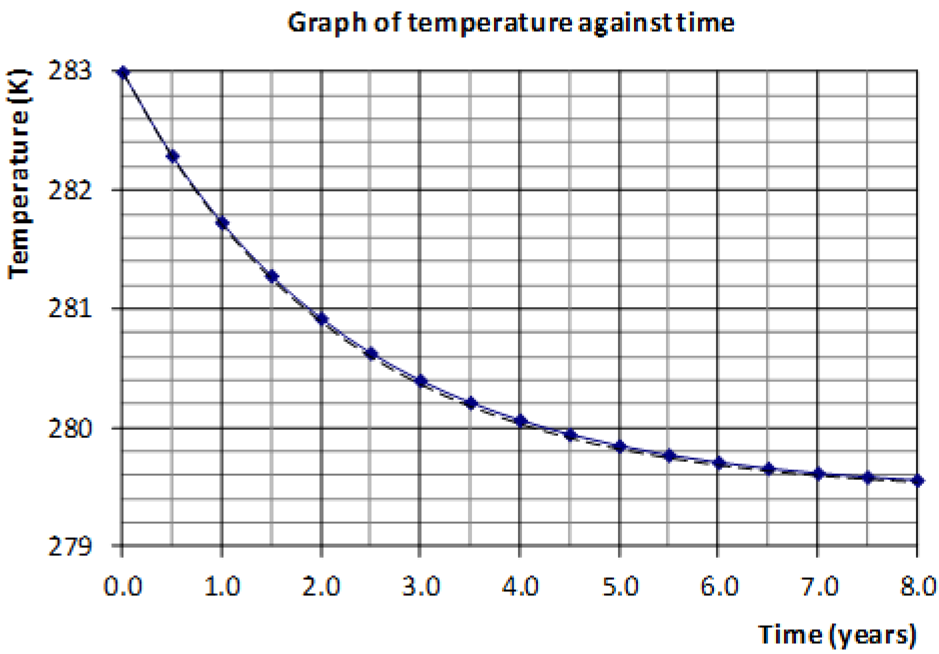
The next graph shows the **cubic** function (dotted line) given by Excel to model the data.



The next graph shows the **quartic** function (dotted line) given by Excel to model the data.



The **exponential** function $y = 279.45 + 3.55e^{-0.45t}$ is shown with the data below.



Again the graphs show that the functions

$y = -0.0114x^3 + 0.2129x^2 - 1.4122x + 283$,
$y = 0.0012x^4 - 0.0298x^3 + 0.2994x^2 - 1.5306x + 283$ and
$y = 279.45 + 3.55e^{-0.45t}$
all give values close to the temperature values for the first 8 years, but the values given by the quadratic function shown below are not so good.
$y = 0.0883x^2 - 1.1047x + 283$

When the graphs are extended to show later years, it can again be seen that only the exponential function gives values likely to model what happens in practice. That is, the temperature approaches a new equilibrium value lower than the original 283 K.

The quadratic model predicts a temperature rise after about 7 years.
The quartic model predicts a temperature rise after 9 years.
The cubic model predicts a steep fall in temperature after 10 years.

Extension answers

Different percentage changes in the incoming energy give different equilibrium temperatures.

If different increments are used (with the same percentage change in the incoming energy) the incremental values of the temperatures are different, but the equilibrium temperature remains the same.